

28. CLOCKS

IMPORTANT FACTS

The Face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called *minute spaces*.

A clock has two hands, the smaller one is called *the hour hand or short hand* while the larger one is called the *minute hand or long hand*.

- i) In 60 minutes, the minute hand gains 55 minutes on the hour hand.
- ii) In every hour, both the hands coincide once.
- iii) The hands are in the same straight line when they are coincident or opposite to each other.
- iv) When the two hands are at right angles, they are 15 minute spaces apart.
- v) When the hands are in opposite directions, they are 30 minute spaces apart.
- vi) Angle traced by hour hand in 12 hrs = 360° .
- vii) Angle traced by minute hand in 60 min. = 360° .

Too Fast and Too Slow: If a watch or a clock indicates 8.15, when the correct time, 8 is said to be 15 minutes too fast.

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

SOLVED EXAMPLES

Ex 1: Find the angle between the hour hand and the minute hand of a clock when 3.25.

Solution: angle traced by the hour hand in 12 hours = 360°

Angle traced by it in three hours 25 min (ie) $41/12$ hrs = $(360 \times 41/12 \times 12)^\circ$
 $= 102 \times 1/2^\circ$

angle traced by minute hand in 60 min. = 360° .

Angle traced by it in 25 min. = $(360 \times 25)/60 = 150^\circ$

Required angle = $1500 - 102 \times 1/2^\circ = 47 \times 1/2^\circ$

Ex 2: At what time between 2 and 3 o'clock will the hands of a clock be together?

Solution: At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, *i.e.* they are 10 min spaces apart.

To be together, the minute hand must gain 10 minutes over the hour hand.

Now, 55 minutes are gained by it in 60 min.

10 minutes will be gained in $(\underline{60} \times 10)/55$ min. = $120/11$ min.

The hands will coincide at $120/11$ min. past 2.

Ex. 3. At what time between 4 and 5 o'clock will the hands of a clock be at right angle?

Sol: At 4 o'clock, the minute hand will be 20 min. spaces behind the hour hand, Now, when the two hands are at right angles, they are 15 min. spaces apart. So, they are at right angles in following two cases.

Case I. When minute hand is 15 min. spaces *behind the hour hand*:

In this case min. hand will have to gain $(20 - 15) = 5$ minute spaces. 55 min. spaces are gained by it in 60 min.

5 min spaces will be gained by it in $\underline{60 \times 5/55}$ min = $60/11$ min.

\therefore They are at right angles at $60/11$ min. past 4.

Case II. When the minute hand is 15 min. spaces *ahead* of the *hour* hand:

To be in this position, the minute hand will have to gain $(20 + 15) = 35$ minute spaces' 55 min. spaces are gained in 60 min.

35 min spaces are gained in $(\underline{60} \times 35)/55$ min = $40/11$

\therefore They are at right angles at $40/11$ min. past 4.

Ex. 4. Find at what time between 8 and 9 o'clock will the hands of a clock being the same straight line but not together.

Sol: At 8 o'clock, the hour hand is at 8 and the minute hand is at 12, *i.e.* the two hands are 20 min. spaces apart.

To be in the same straight line but not together they will be 30 minute spaces apart.

So, the minute hand will have to gain $(30 - 20) = 10$ minute spaces over the hour hand.

55 minute spaces are gained. in 60 min.

10 minute spaces will be gained in $(60 \times 10)/55$ min. = $120/11$ min.

∴ The hands will be in the same straight line but not together at $120/11$ min.

Ex. 5. At what time between 5 and 6 o'clock are the hands of a clock 3 min apart?

. **Sol.** At 5 o'clock, the minute hand is 25 min. spaces behind the hour hand.

Case I. Minute hand is 3 min. spaces *behind the hour hand*.

In this case, the minute hand has to gain' $(25 - 3) = 22$ minute spaces. 55 min. are gained in 60 min.

22 min. are gained in $(60 \times 22)/55$ min. = 24 min.

∴ The hands will be 3 min. apart at 24 min. past 5.

Case II. Minute hand is 3 min. spaces *ahead of the hour hand*.

In this case, the minute hand has to gain $(25 + 3) = 28$ minute spaces. 55 min. are gained in 60 min.

28 min. are gained in $(60 \times 28)/55 = 346/11$

The hands will be 3 min. apart at $346/11$ min. past 5.

Ex 6. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Sol: In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand. 55 min. are gained in 60 min.

60 min are gained in $\frac{60 \times 60}{55}$ min = $720/11$ min.

But, they are together after 65 min.

Gain in 65 min = $720/11 - 65 = 5/11$ min.

Gain in 24 hours = $(5/11 * (60 * 24)/65)$ min = $440/43$

The clock gains $\frac{440}{43}$ minutes in 24 hours.

Ex. 7. A watch which gains uniformly, is 6 min. slow at 8 o'clock in the morning Sunday and it is 6 min. 48 sec. fast at 8 p.m. on following Sunday. When was it correct?

Sol. Time from 8 a.m. on Sunday to 8 p.m. on following Sunday = 7 days 12 hours
= 180 hours

The watch gains $(5 + \frac{29}{5})$ min. or $\frac{54}{5}$ min. in 180 hrs.
Now $\frac{54}{5}$ min. are gained in 180 hrs.

5 min. are gained in $(180 \times \frac{5}{54} \times 5)$ hrs. = 83 hrs 20 min. = 3 days 11 hrs 20 min.

Watch is correct 3 days 11 hrs 20 min. after 8 a.m. of Sunday.
It will be correct at 20 min. past 7 p.m. on Wednesday.

Ex 8. A clock is set right at 6 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?

Sol. Time from 5 a.m. on a day to 10 p.m. on 4th day = 89 hours.
Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

$\frac{356}{15}$ hrs of this clock = 24 hours of correct clock.

89 hrs of this clock = $(24 \times \frac{31556}{15} \times 89)$ hrs of correct clock.
= 90 hrs of correct clock.

So, the correct time is 11 p.m.

Ex. 9. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 p.m. on the following day?

Sol. Time from 8 a.m. on a day 1 p.m. on the following day = 29 hours.
24 hours 10 min. of this clock = 24 hours of the correct clock.

$\frac{145}{6}$ hrs of this clock = 24 hrs of the correct clock

29 hrs of this clock = $(24 \times \frac{6}{145} \times 29)$ hrs of the correct clock

= 28 hrs 48 min. of correct clock

The correct time is 28 hrs 48 min. after 8 a.m.

This is 48 min. past 12.

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